Instantaneous Reaction-Time in Dynamic-Consistency Checking of Conditional Simple Temporal Networks

Massimo Cairo　*Carlo Comin*　*Romeo Rizzi*

University of Trento
Trento, Italy

University of Verona
Verona, Italy

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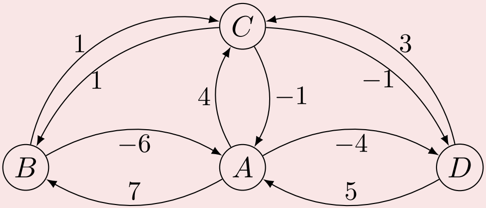
Simple Temporal Networks (STNs) [Dechter, Meiri, Pearl [3]]

- Represent a general framework for analyzing systems (conjunctions) of difference constraints on ordered pairs of temporal variables.
- An STN can be encoded by a weighted directed graph:
  - a node represents a time-point variable (time-point);
  - an arc represents a temporal distance constraint:
    - ★ $u \stackrel{a}{\to} v$ stands for $v - u \leq a$, $a \in$ R;
    - ★ $(v \stackrel{-a}{\to} u$ stands for $v - u \geq a)$.



Represented Constraints:

$4 \leq A - D \leq 5$
$6 \leq B - A \leq 7$
$1 \leq C - A \leq 4$
$-1 \leq B - C \leq 1$
$1 \leq C - D \leq 3$

Introduction - STNs

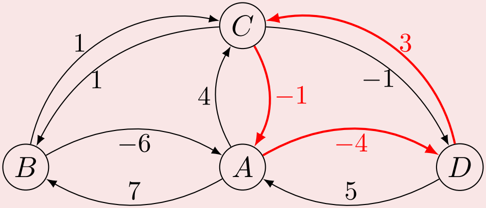
STN Consistency [Dechter, Meiri, Pearl [3]]

- An STN $\langle G = (V, E), \ell \rangle$ is consistent if it admits a feasible scheduling function, i.e., we can assign a real value $s(v)$ to each time-point v, such that all constraints are satisfied:

  $\exists\, s : V \mapsto \mathbb{R}$ such that:

$$s(v) \leq s(u) + \ell_{(u,v)} \qquad \forall (u, v) \in E.$$

- An STN is not consistent if it contains a negative cycle.



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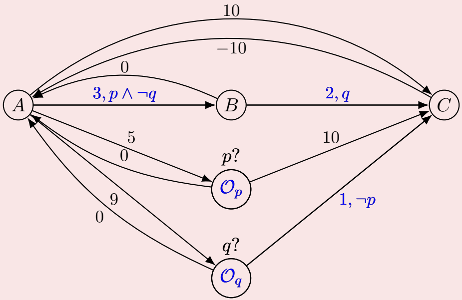
Conditional Simple Temporal Networks: the Model

CSTN $\Gamma = \langle V, \mathcal{A}, L, \mathcal{O}, \mathcal{O}V, P \rangle$ [Tsamardinos, Vidal, Pollack [5] / Hunsberger, Posenato, Combi [4]

- The CSTN formalism extends STNs in that:
  1. some of the nodes are called **observation events** and to each of them is associated a boolean variable, to be disclosed only at execution time;
  2. **labels** (i.e. conjunctions over the literals) are attached to all nodes *and* constraints, to indicate the situations in which each of them is required.



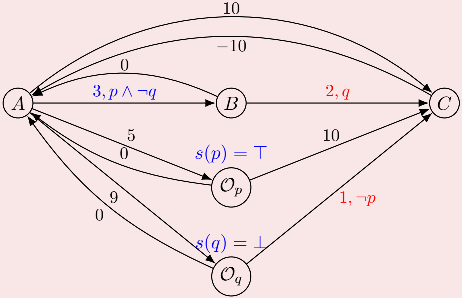
Conditional Simple Temporal Networks: the Model

Scenario

A **scenario** s over a set P of boolean variables is a truth assignment:

$$s : P \to \{\top, \bot\}.$$

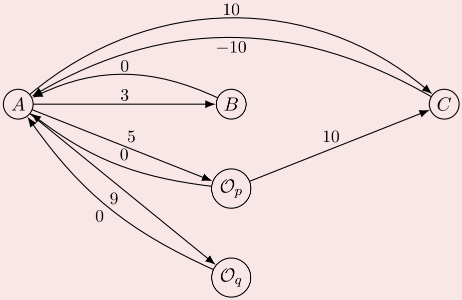
Let $s(p) = \top$ and $s(q) = \bot$...



Conditional Simple Temporal Networks: the Model

The restriction STN Γ_s^+.

- The **restriction** of V and A w.r.t. the scenario $s \in \Sigma_P$ are:
  - $V_s^+ \triangleq \{v \in V \mid s(L(v)) = \top\}$;
  - $A_s^+ \triangleq \{\langle u, v, w\rangle \mid \exists \ell \, \langle v - u \leq w, \ell\rangle \in A, s(\ell) = \top\}$.
- The **restriction** of Γ w.r.t. s is the STN $\Gamma_s^+ \triangleq \langle V_s^+, A_s^+\rangle$.

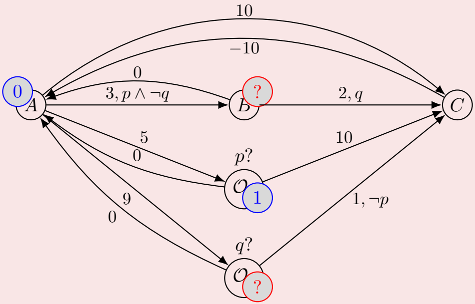


Scheduling

A **scheduling** for a subset of events $U \subseteq V$ is a function:

$$\phi : U \to \mathbf{R},$$

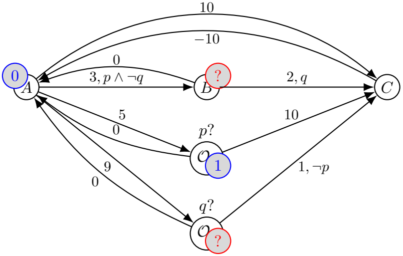
that assigns a real number to each event in U.

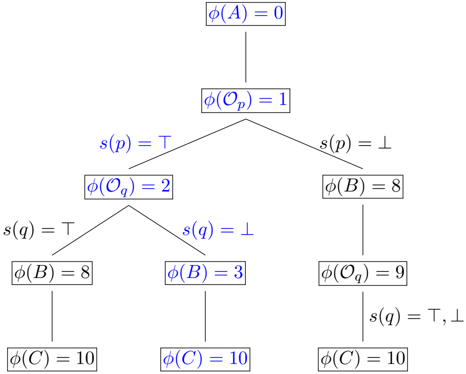


CSTN's Consistencies

Three notions of consistency arise for CSTNs: *weak*, *strong*, and...

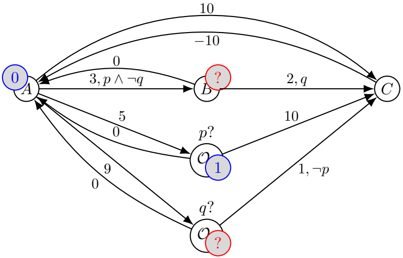
dynamic consistency.

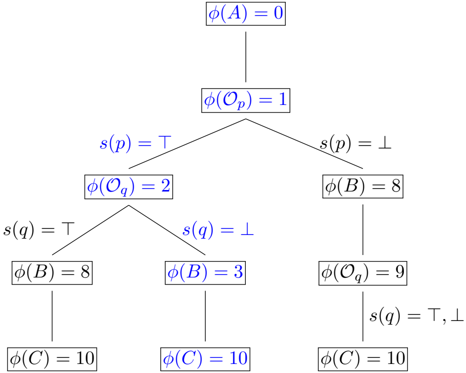




CSTN's Dynamic Consistency

Dynamic Consistency (DC) requires the existence of conditional plans where **decisions** about precise timing of actions **are postponed until exec. time**, but it guarantees that all the relevant constraints will be ultimately satisfied.

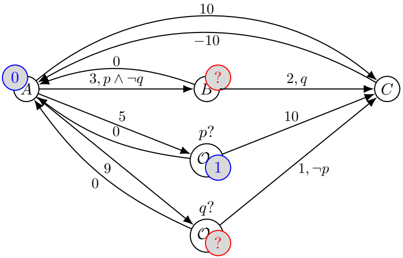


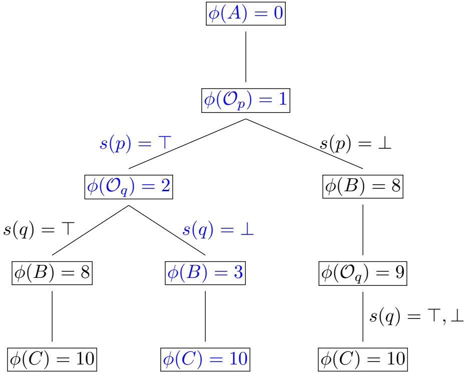


CSTNs: Consistency

Execution Strategy

An **Execution Strategy** for Γ is a mapping $\sigma : \Sigma_P \to \Phi_V$ such that, for any scenario $s \in \Sigma_P$, the domain of the scheduling $\sigma(s) \in \Phi_V$ is V_s^+.





Difference Set $\Delta(s_1; s_2)$

$$\Delta(s_1; s_2) \triangleq \Big\{\mathcal{O}_p \in V^+_{s_1} \cap \mathcal{O}V \mid s_1(p) \neq s_2(p)\Big\}.$$

Dynamic Execution Strategy

Let $\sigma \in \mathcal{S}_\Gamma$ be an execution strategy. Then, σ is **dynamic** if and only if the following implication holds for every $s_1, s_2 \in \Sigma_P$, $u \in V^+_{s_1,s_2}$:

$$\Big(\bigwedge_{v \in \Delta(s_1;s_2)} [\sigma(s_1)]_u \leq [\sigma(s_1)]_v \Big) \Rightarrow [\sigma(s_1)]_u = [\sigma(s_2)]_u$$

Our previous contribution at TIME2015

A (pseudo) Singly-Exponential Time DC-Checking for CSTNs [CR, TIME 2015]

- There exists a *(pseudo)* **singly-exponential** time algorithm for checking **DC** on any input CSTN $\Gamma = \langle V, A, L, \mathcal{O}, \mathcal{O}V, P \rangle$.
  - Here, $W \triangleq \max_{a \in A} |w_a|$ and $|\Sigma_P| \leq 2^{|P|}$.
- In particular, given any dynamically-consistent CSTN Γ, the algorithm returns a viable and dynamic execution strategy.

Technique [CR, TIME 2015]

Reduction from *CSTN-DC-Checking* to *Mean Payoff Games* and *Energy Games*.

ϵ-Dynamic Consistency and the Reaction Time $\hat{\epsilon}(\Gamma)$ [CR, TIME 2015]

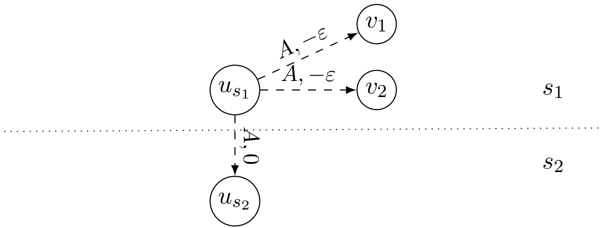
In order to analyze the algorithm, we introduced a novel and refined notion of dynamic-consistency, named ϵ**-dynamic-consistency**;

ϵ-dynamic-consistency [CR, TIME2015]

Given any CSTN $\langle V, A, L, \mathcal{O}, \mathcal{O}V, P\rangle$ and any real number $\epsilon \in (0, +\infty)$, an execution strategy $\sigma \in \mathcal{S}_\Gamma$ is ***ϵ-dynamic*** if, for any two scenarios $s_1, s_2 \in \Sigma_P$ and any event $u \in V^+_{s_1,s_2}$:

$$[\sigma(s_1)]_u \geq \min\Big(\{[\sigma(s_2)]_u\} \cup \{[\sigma(s_1)]_v + \epsilon \mid v \in \Delta(s_1; s_2)\}\Big)$$

A CSTN Γ is ***ϵ-DC*** if it admits $\sigma \in \mathcal{S}_\Gamma$ which is both *viable* and *ϵ-dynamic*.



ϵ-dynamic-consistency and reaction-time $\hat{\epsilon}(\Gamma)$

Reaction-Time $\hat{\varepsilon}(\Gamma)$

- This allowed us to provide a sharp lower bounding analysis of the critical value of the **reaction time** $\hat{\varepsilon}(\Gamma)$ where the CSTN Γ transits from being, to not being, dynamically-consistent.
  - Here, $\hat{\epsilon}(\Gamma) \triangleq \sup\big\{\epsilon \in \mathbf{R}_{>0} \mid \Gamma$ is ϵ-dynamically-consistent$\big\}$.

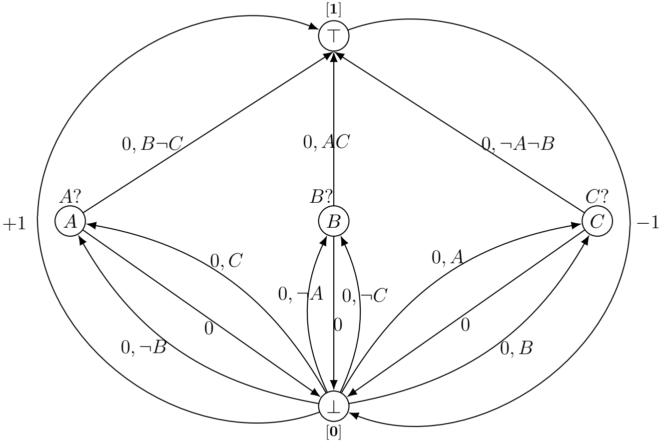
Instantaneous Reaction-Time

- However, the ε-DC notion is interesting per se, and the ε-DC-Checking algorithm (TIME2015) rests on the assumption that $\varepsilon > 0$;
  - i.e., leaving unsolved the question of what happens when $\varepsilon = 0$.
- In this work (TIME2016) we introduce and study π-DC, a sound notion of DC with an **instantaneous** reaction-time.
  - i.e., one in which the planner can react to any observation *at the same instant of time* in which the observation is made.

ϵ-DC$_{|_{\epsilon=0}}$ and π-DC

Are ϵ-DC$_{|_{\epsilon=0}}$ and π-DC not the same ?

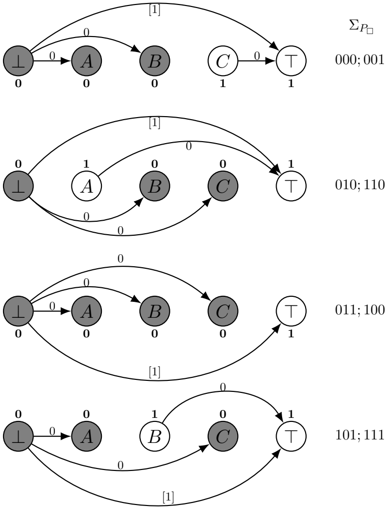
Consider the following CSTN $\Gamma_\Box$.



The CSTN $\Gamma_\Box$ is 0-DC.

ϵ-DC$|_{\epsilon=0}$ and π-DC

Proof:



ϵ-DC$_{|_{\epsilon=0}}$ and π-DC

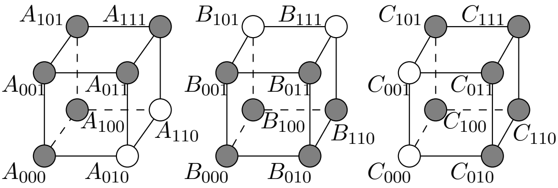


Figure: The ES $\sigma_\square$ for the CSTN $\Gamma_\square$.

The CSTN $\Gamma_\square$ is not DC.

- No 3-cube is completely black.

π-DC

We need to take explicitly into account an *additional ordering* between the observation events scheduled *at the same execution time*.

π-Execution-Strategy

An *ordered-Execution-Strategy (π-ES)* for Γ is a mapping:

$$\sigma : s \mapsto \left([\sigma(s)]^t, [\sigma(s)]^\pi\right),$$

$s \in \Sigma_P$, $[\sigma(s)]^t \in \Phi_V$ and $[\sigma(s)]^\pi : \mathcal{O}V_s^+ \rightleftharpoons \{1, \ldots, |\mathcal{O}V_s^+|\}$ is bijective.

Remark

We require positions to be *coherent* w.r.t. execution times, i.e., $\forall(\mathcal{O}_p, \mathcal{O}_q \in \mathcal{O}V_s^+)$ if $[\sigma(s)]^t_{\mathcal{O}_p} < [\sigma(s)]^t_{\mathcal{O}_q}$ then $[\sigma(s)]^\pi_{\mathcal{O}_p} < [\sigma(s)]^\pi_{\mathcal{O}_q}$.

π-DC

π-History

Let $\sigma \in \mathcal{S}_\Gamma$, $s \in \Sigma_P$, and let $\tau \in \mathbf{R}$ and $\psi \in \{1, \ldots, |V|\}$.
The *ordered-history* π-Hst(τ, ψ, s, σ) of τ and ψ in the scenario s, under the π-ES σ, is defined as: π-Hst$(\tau, \psi, s, \sigma) =$
$= \Big\{ (p, s(p)) \in P \times \{0, 1\} \mid \mathcal{O}_p \in \mathcal{O}V_s^+, [\sigma(s)]^t_{\mathcal{O}_p} \leq \tau, [\sigma(s)]^\pi_{\mathcal{O}_p} < \psi \Big\}$.

π-Dynamic-Consistency

Any $\sigma \in \mathcal{S}_\Gamma$ is called *π-dynamic* when, for any two scenarios $s_1, s_2 \in \Sigma_P$ and any event $u \in V^+_{s_1,s_2}$, if $\tau \triangleq [\sigma(s_1)]^t_u$ and $\psi \triangleq [\sigma(s_1)]^\pi_u$, then:

$$\pi\text{-Hst}(\tau, \psi, s_1, \sigma) \wedge s_2 \Rightarrow [\sigma(s_2)]^t_u = \tau, [\sigma(s_2)]^\pi_u = \psi.$$

We say that Γ is *π-dynamically-consistent (π-DC)* if it admits $\sigma \in \mathcal{S}_\Gamma$ which is both viable and π-dynamic. The problem of checking whether a given CSTN is π-DC is named π-DC-Checking.

π-DC

Remark

Due to the strict inequality "$[\sigma(s)]^{\pi}_{\mathcal{O}_p} < \psi$" in the definition of π-Hst$(\cdot)$, in a π-dynamic π-ES, there must be exactly one $\mathcal{O}_{p'} \in \mathcal{O}V$, for some $p' \in P$, which is executed at first (w.r.t. both execution time and position) under all possible scenarios $s \in \Sigma_P$ (i.e., a root).

The CSTN $\Gamma_{\square}$ was not π-DC:

- There was no root observation.

π-DC

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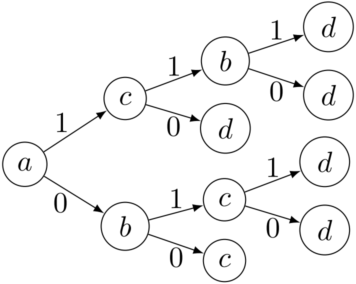
Due to the strict inequality "$[\sigma(s)]^{\pi}_{\mathcal{O}_p} < \psi$" in the definition of π-Hst$(\cdot)$, in a π-dynamic π-ES, there must be exactly one $\mathcal{O}_{p'} \in \mathcal{O}V$, for some $p' \in P$, which is executed at first (w.r.t. both execution time and position) under all possible scenarios $s \in \Sigma_P$ (i.e., a root).



An example of a *permutation-scenario-tree (ps-tree)* over $P = \{a, b, c, d\}$.

π-DC $\not\Rightarrow$ 0-DC

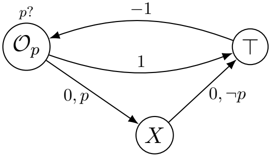


Figure: The CSTN Γ_π.

π-DC $\not\Rightarrow$ 0-DC

The CSTN Γ_π is π-DC, but it is not DC.

- $t(\mathcal{O}_p) = 0$; if $s(\mathcal{O}_p) = \texttt{true}$, then $t(X) = 0$; else, $t(X) = 1$.
- Still, X can't be scheduled ϵ-dynamically, for any $\epsilon > 0$.

Summary of CSTN-DC notions

In summary, the following chain of implications hold on the various DCs:

$$\boxed{(\epsilon\text{-DC}, \epsilon > 0) \Leftrightarrow \text{DC} \overset{\not\Leftarrow}{\Rightarrow} \pi\text{-DC} \overset{\not\Leftarrow}{\Rightarrow} (\epsilon\text{-DC}, \epsilon = 0)}$$

A Singly-Exp Time Algorithm for π-DC-Checking of CSTNs

Singly-Exp Time Algorithm for π-DC-Checking [CCR, TIME2016]

There exists an algorithm for checking π-DC on any input given CSTN $\Gamma = (V, A, L, \mathcal{O}, \mathcal{O}V, P)$ with the following *(pseudo) singly-exponential* time complexity:

$$O\Big(|\Sigma_P|^4|A|^2|V|^3 + |\Sigma_P|^5|A||V|^4|P| + |\Sigma_P|^6|V|^5|P|\Big)W.$$

Moreover, when Γ is π-DC, the algorithm also returns a viable and π-dynamic π-ES for Γ. Here, $W \triangleq \max_{a\in A} |w_a|$.

Technique

A **reduction** from π*-DC-Checking* to *DC-Checking* is identified.

- Then one reduces to *Mean Payoff Games / Energy Games* [CR, TIME2015].

A Singly-Exp Time Algorithm for π-DC-Checking of CSTNs

Idea of the reduction

- Give a small margin $\gamma \in (0, 1)$ so that the planner can do before, in the sense of the time value $[\sigma(s)]_v$, what he did “before" in the ordering π.
- Given any ES in the relaxed network, the planner would then turn it into a π-ES for the original network (which has some more stringent constraints), by rounding-down each time value $[\sigma(s)]_v$ to the largest integer less than or equal to it, i.e., $\lfloor [\sigma(s)]_v \rfloor$.
- **Problem:** one may (possibly) violate some constraints when there is a “leap" in the rounding (i.e., a difference of one unit, in the rounded value, w.r.t. what one would have wanted).
  - We have identified a technique to get around this subtle case, provided that γ is exponentially small.

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A Singly-Exp Time Algorithm for π-DC-Checking of CSTNs

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A Singly-Exp Time Algorithm for π-DC-Checking of CSTNs

Relaxed CSTN Γ'_γ, for any $\gamma \in (0,1)$.

- Let $\Gamma = \langle V, A, L, \mathcal{O}, \mathcal{O}V, P\rangle$ be any CSTN with integer constraints.
- Let $\gamma \in (0,1)$ be a real.
- Define $\Gamma'_\gamma \triangleq \langle V, A'_\gamma, L, \mathcal{O}, \mathcal{O}V, P\rangle$ to be a CSTN that differs from Γ only in the numbers appearing in the constraints.
  - Specifically, each constraint $\langle u - v \leq \delta, \ell\rangle \in A$ is replaced in Γ'_γ by a slightly relaxed constraint, $\langle u - v \leq \delta'_\gamma, \ell\rangle \in A'_\gamma$, where:

$$\delta'_\gamma \triangleq \delta + |V| \cdot \gamma.$$

Lemma (Easy Direction).

Let γ be any real in $(0, |V|^{-1})$. If Γ is π-DC, then Γ'_γ is DC.

Proof (Sketch):

- Since Γ is π-DC, $\exists$ an *integral*, viable and π-dynamic, π-ES σ for Γ.
- Fix some real $\gamma \in (0, |V|^{-1})$.
- Define the ES $\sigma'_\gamma \in \mathcal{S}_{\Gamma'_\gamma}$ as follows, for every $s \in \Sigma_P$ and $v \in V_s^+$:

$$[\sigma'_\gamma(s)]_v \triangleq [\sigma(s)]_v^t + [\sigma(s)]_v^\pi \cdot \gamma.$$

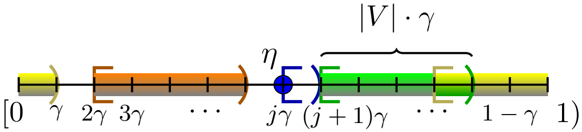
- Then, one can prove that σ'_γ is *viable* and *dynamic* for Γ'_γ.

Lemma (Hard Direction).

Let γ be any real in $(0, |\Sigma_P|^{-1} \cdot |V|^{-2})$. If Γ'_γ is DC, then Γ is π-DC.

Proof (Sketch):

1. Let $\sigma'_\gamma \in \mathcal{S}_{\Gamma'_\gamma}$ be some viable and dynamic ES for Γ'_γ.
2. Pick $\eta \in [0, 1)$ such that:
   $[\sigma'_\gamma(s)]_v - \eta - k \in [0, |V| \cdot \gamma)$, for *no* $v \in V, s \in \Sigma_P, k \in \mathbf{Z}$.
   - such a value η exists. Indeed, there are only $|\Sigma_P| \cdot |V|$ choices of pairs $(s, v) \in \Sigma_P \times V$ and each pair rules out a (circular) semi-open interval of length $|V| \cdot \gamma$ in $[0, 1)$, so the total measure of invalid values for η in the semi-open real interval $[0, 1)$ is at most $|\Sigma_P| \cdot |V| \cdot |V| \cdot \gamma < 1$.



A Singly-Exp Time Algorithm for π-DC-Checking of CSTNs

Lemma (Hard Direction).

Let γ be any real in $(0, |\Sigma_P|^{-1} \cdot |V|^{-2})$. If Γ'_γ is DC, then Γ is π-DC.

Proof (Sketch):

3. By subtracting η to all time values, we can assume w.l.o.g. that:

$$[\sigma'_\gamma(s)]_v - \lfloor [\sigma'_\gamma(s)]_v \rfloor \geq |V| \cdot \gamma, \text{ for } \textit{all } s \in \Sigma_P \text{ and } v \in V_s^+.$$

4. Let $[\sigma(s)]_v^t \triangleq \lfloor [\sigma'_\gamma(s)]_v \rfloor$, and $[\sigma(s)]^\pi$ be the ordering induced by $\sigma'_\gamma(s)$.
5. Then, one can prove that $([\sigma(s)]^t, [\sigma(s)]^\pi)$ is *viable* and π-*dynamic*.

A Singly-Exp Time Algorithm for π-DC-Checking of CSTNs

Main Theorem

Let Γ be a CSTN and let $\gamma \in (0, |\Sigma_P|^{-1} \cdot |V|^{-2})$; *e.g.* $\gamma \triangleq \frac{1}{|\Sigma_P| \cdot |V|^2 + 1}$.
Then, Γ is π-DC if and only if Γ'_γ is DC.

(From Lemma (Hard Direction), one can actually compute a π-dynamic and viable π-ES for Γ)

Thank you for your attention.

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